Workshop: quantum algorithms for asymmetric cryptography

Greg Meyer [1]

[1]MIT

July 9, 2026

Most important open problem: why we are all here

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Let me know if you have ideas; we can vote on Friday.

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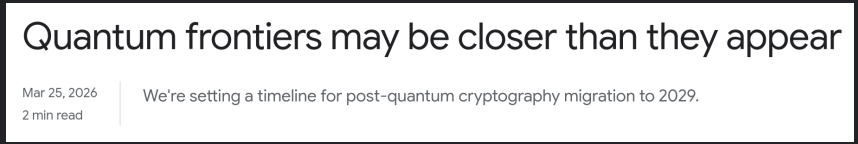
We’re on a timeline to figure out a name because it will be dead soon…

Why we are actually here: quantum attacks are coming

Google:



Cloudflare:

Why we are actually here: quantum attacks are coming

US government:

Why we are actually here: quantum attacks are coming

As experts in those quantum attacks, what do we do about it?

Roadmap

Goals of the workshop

Roadmap

Goals of the workshop

Landscape of vulnerable cryptography

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Some history: how did we get here?

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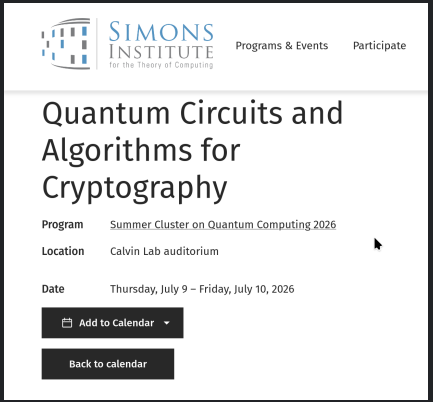
Workshop goals

Discuss the *operational* status of quantum cryptanalysis



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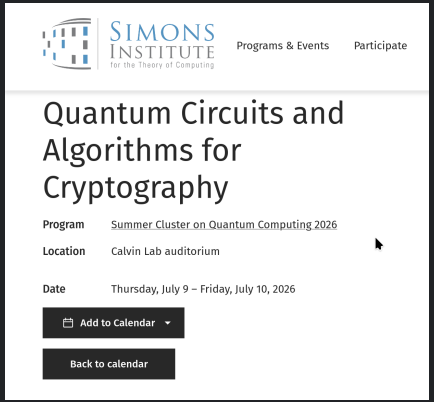

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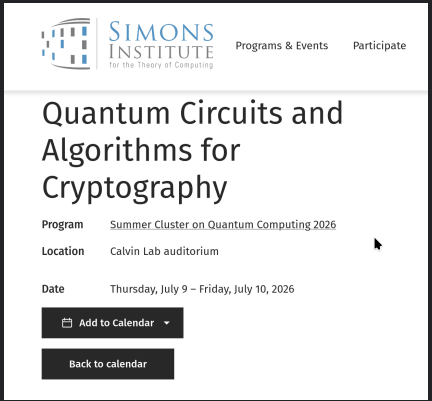

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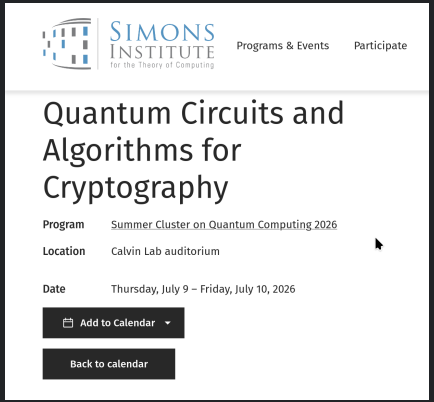

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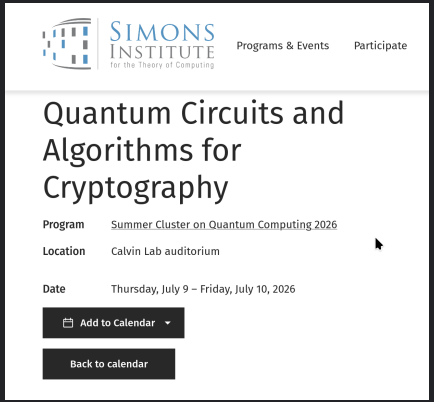

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Don't be afraid to interrupt with questions, ideas, etc.!

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	Thursday	Friday
9:30-10:30	Speaker: Greg Meyer	Speaker: Craig Gidney
11:00-12:00	Speaker: Martin Ekerå	Speaker: André Schrottenloher
2:00-3:00	Speaker: Seyoon Ragavan	Panel 2: *Responsible disclosure*
3:30-5:00	Panel 1: *Open directions*	Lightning talks/discussion

Panel 1: Open directions

- Martin Ekerå (Swedish NCSA and KTH)
- Craig Gidney (Google)
- André Schrottenloher (INRIA)

Panel 2: Responsible disclosure

- Ryan Babbush (Google)
- Rolfe Schmidt (Signal)
- Martin Ekerå (Swedish NCSA and KTH)
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Landscape of vulnerable cryptography

Some history: how did we get here?

Abstract computational problems

Discrete logarithm

Consider a multiplicative group $\mathbb{G}$ with generator g.

Given a group element $x \in \mathbb{G}$, **find d such that $g^d = x$.**

Integer factorization

Consider an integer N.

Find its prime decomposition.

Vulnerable concrete problems used in cryptography

Elliptic curve discrete log
(ECDLP)

For an elliptic curve group $\mathbb{G}$, consider two curve points g and $x = [d]g$.

Find d.

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RSA integer factorization
(IFP)

Consider an integer $N = pq$, where p, q are primes of equal bit length.

Find p **and** q**.**

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Note: cryptosystems rely on slightly stronger hardness assumptions

Comparing the vulnerable problems

Note: this entire slide is my qualitative assessment, in July 2026

IFP = integer factorization; ECDLP = elliptic curve discrete log; FFDLP = finite field discrete log

Attention given	How widely used?	Quantum cost?

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Takeaway: factoring gets by far the *most attention*, but not seemingly for any *good reason*.

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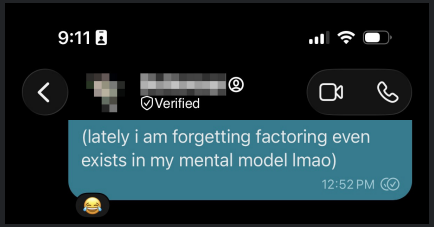
A window into my scientific process



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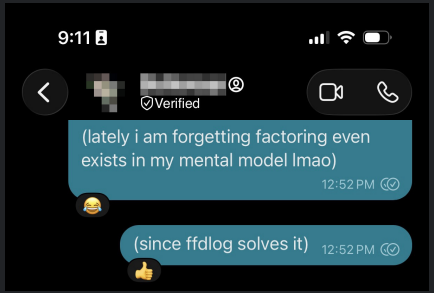

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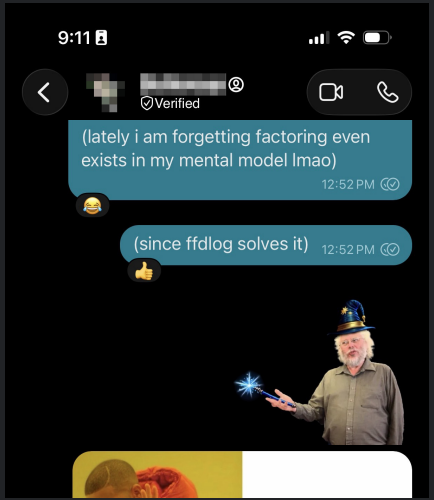

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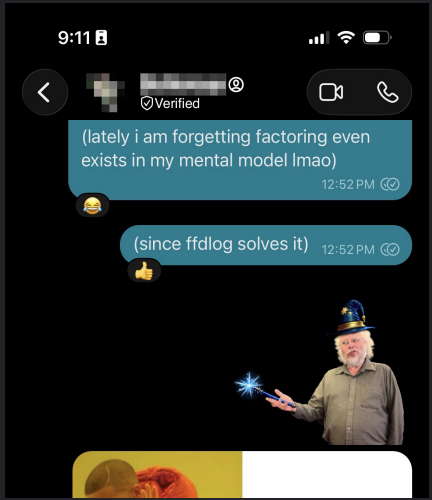

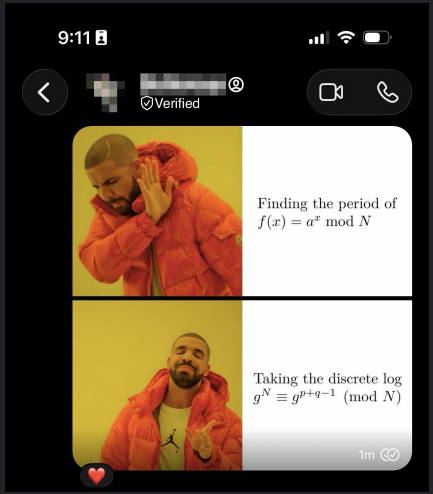

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→ You will hear about this reduction (and more) from Martin
during his **11 AM-12 PM talk today**!

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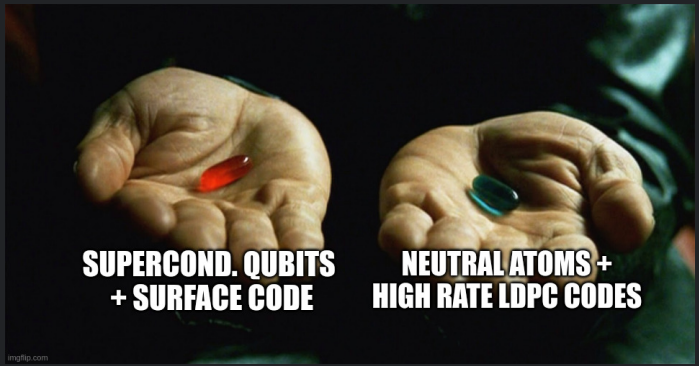

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Surface code
+ superconducting

High rate LDPC
+ neutral atoms

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Surface code
+ superconducting

- Fast cycle time

High rate LDPC
+ neutral atoms

- Slow cycle time

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Surface code
+ superconducting

- Fast cycle time
- Many physical qubits per logical

High rate LDPC
+ neutral atoms

- Slow cycle time
- Fewer physical qubits per logical

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Metrics for cost

How should we balance logical *qubits* vs *Toffolis*? (not to mention depth, other gates, ...)

Depends strongly on architecture!

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Many algorithmic decisions made with **surface code in mind.**
Take a second look at old constructions under new heuristics?

Quantum cost: rough overview

Elliptic curve discrete log (ECDLP)

Given $x = [d]g$, find d.

$n = \lceil \log_2 |\mathbb{G}| \rceil$

Qubits: $\sim 4.4n + O(\sqrt{n})$
Gates: $\widetilde{O}(n^3)$

For $n = 256$:

- Qubits: ~ 1200
- Toffolis: $\sim 2^{26}$

Finite field discrete log (FFDLP)

Given $x \equiv g^d \pmod{p}$ find d.

$n = \lceil \log p \rceil \quad m = \lceil \log d \rceil$

Qubits: $m + o(m) + O(\log n)$
Gates: $\widetilde{O}(mn^2)$

For $n = 2048$, $m = 225$:

- Qubits: ~ 350
- Toffolis: $\sim 2^{26}$

RSA integer factorization (IFP)

Given $N = pq$, find p and q.

$n = \lceil \log_2 N \rceil$

$\Leftarrow$ FFDLP with params.
$n = \lceil \log_2 N \rceil$, $m = n/2$

For $n = 2048$:

- Qubits: ~ 1400
- Toffolis: $\sim 2^{33}$

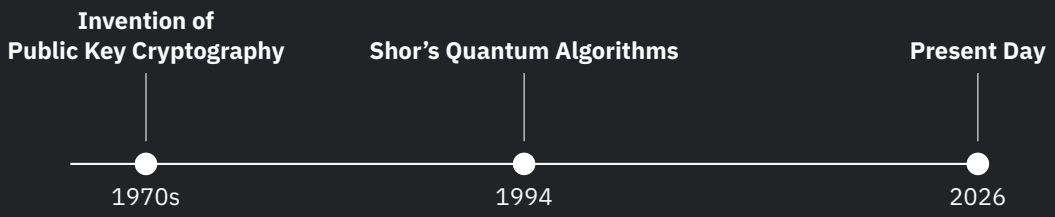
Roadmap

Goals of the workshop

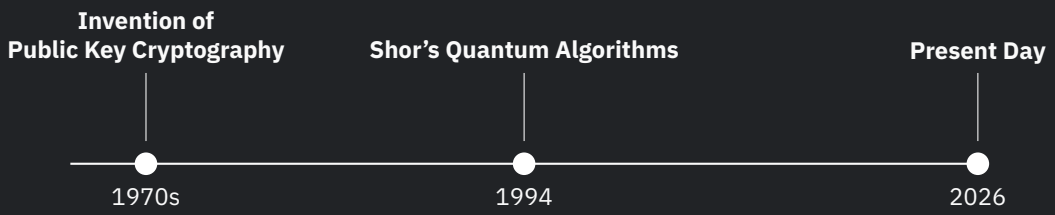
Landscape of vulnerable cryptography

Some history: how did we get here?

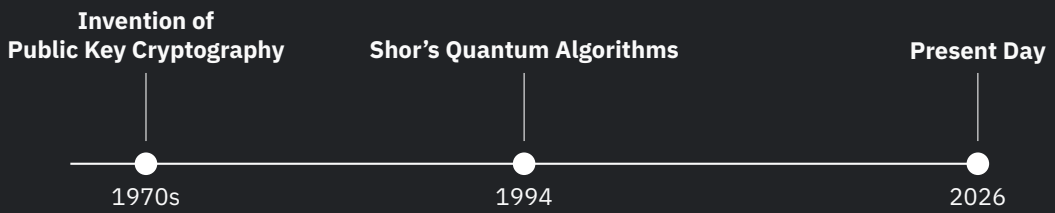
Timeline of public-key cryptography



Timeline of public-key cryptography

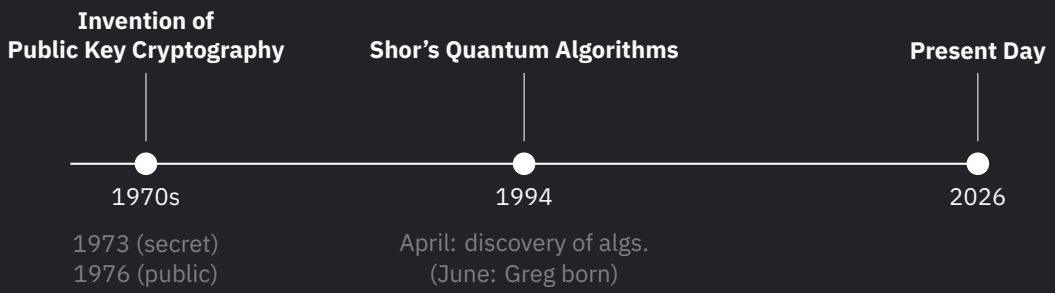


Timeline of public-key cryptography



Happy 32nd birthday to Shor's algorithms! 🥳

Timeline of public-key cryptography



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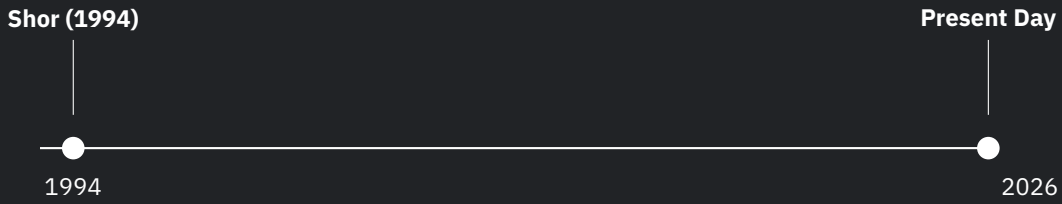
Shor's discovery was closer to the invention of public-key cryptography than it is to the present day

Timeline of quantum circuits for discrete log + factoring



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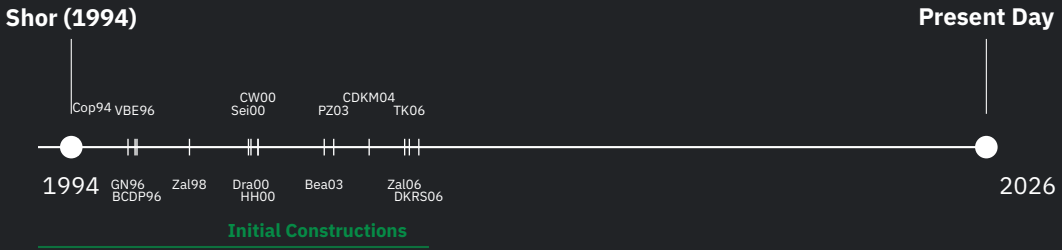

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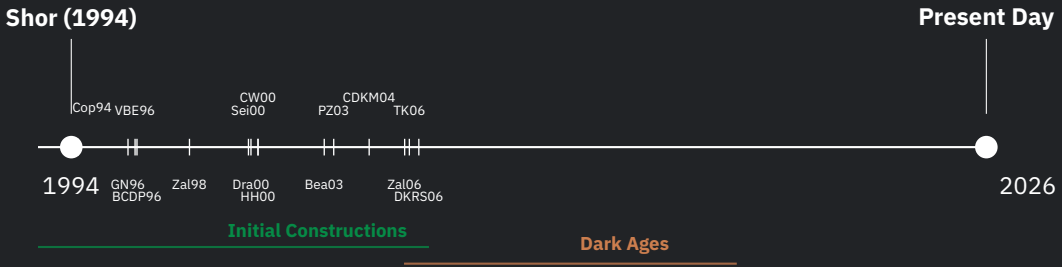

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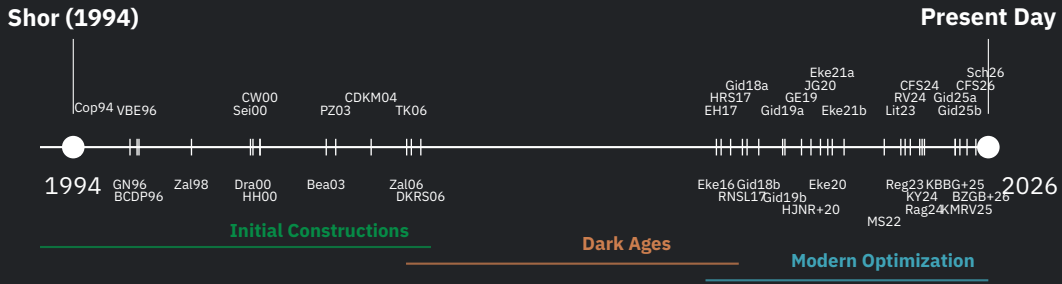

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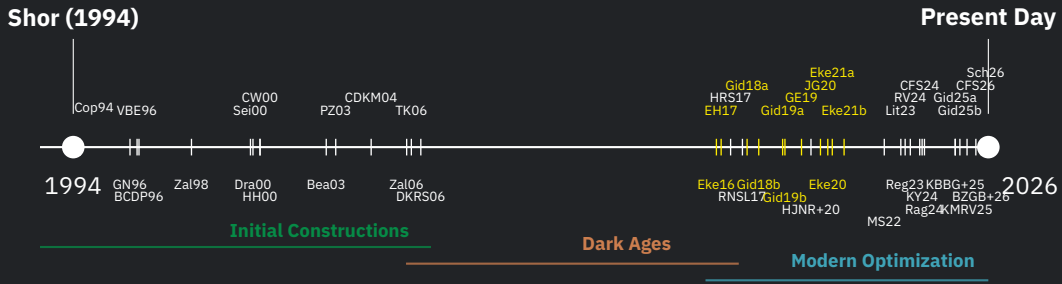

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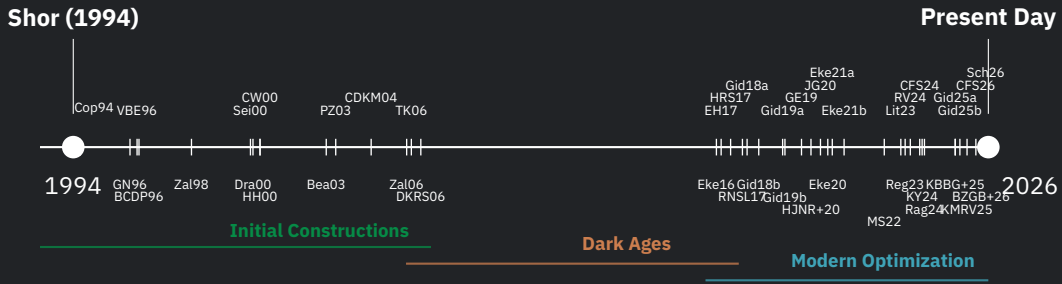

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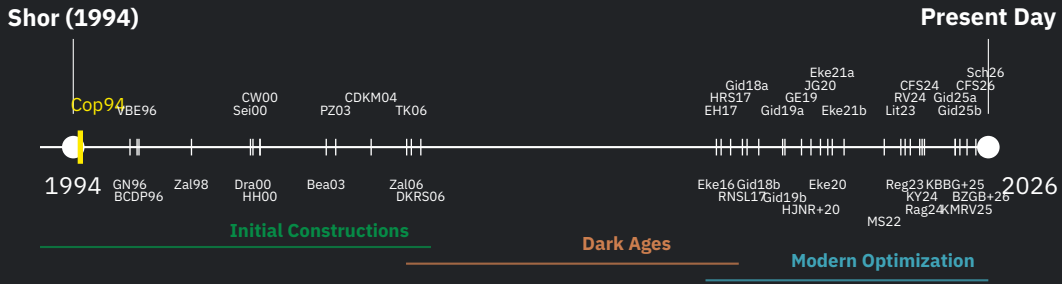
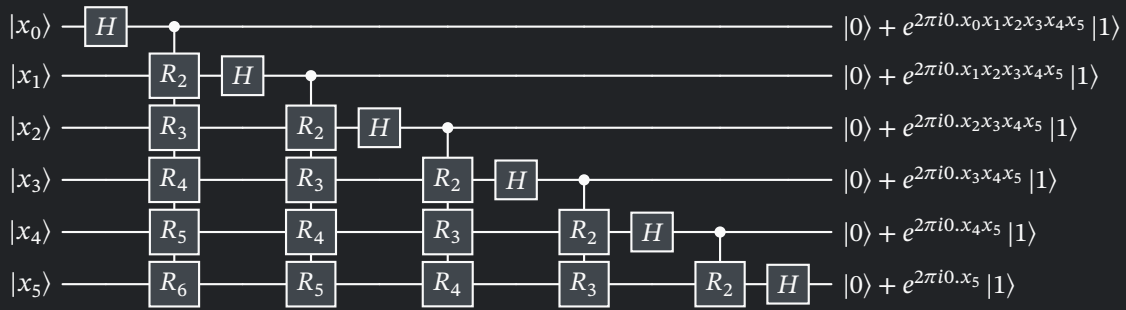


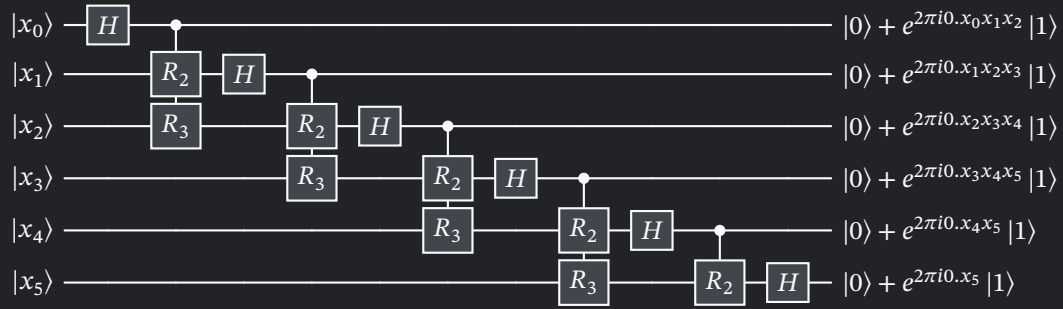
Coppersmith ’94: approximate quantum Fourier transform modulo 2^n

This is the “standard” QFT construction



Coppersmith ’94: approximate QFT_{2^n}

Approximate QFT: drop small rotations!



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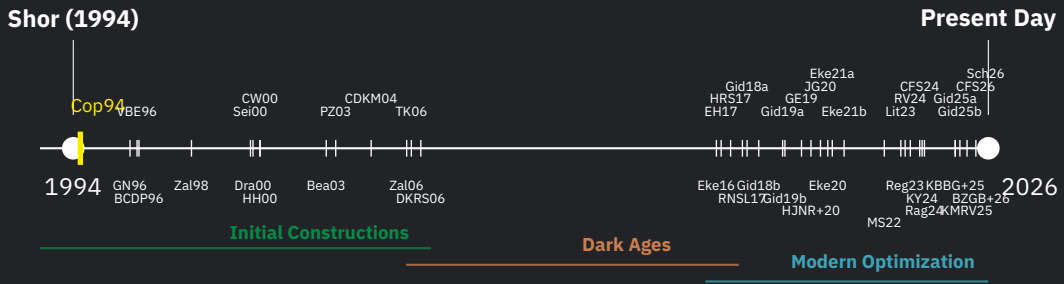

Peter Shor didn't have the now-standard QFT_{2^n} construction for his original paper!

Peter Shor’s original quantum Fourier transform (QFT)

Crucial question: can we efficiently perform the Fourier transform on the amplitudes of an n-qubit quantum state?

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Observation:

- QFT_m for $m \lesssim O(\log n)$ is doable by "brute force" in $\mathrm{poly}(n)$ time

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- Maybe we can combine a bunch of tiny Fourier transforms into one big one?

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- Maybe we can combine a bunch of tiny Fourier transforms into one big one?
  - This is possible via a **Residue Number System (RNS)**.

Aside: residue number systems (RNS)

A funky and neat alternative to binary representation!

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A funky and neat alternative to binary representation!

Note: You will hear a lot more about RNS from André tomorrow at 11 AM!

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A funky and neat alternative to binary representation!

To represent an n-bit integer x:

1. Pick a bunch of co-prime integers $\{m_i\}$, each of length $O(\log n)$

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Fact: *(Chinese Remainder Theorem)*

If $x < L$:

$$\sum_i x_i F_i \bmod L = x$$

(where F_i are some pre-computable integers that don't depend on x).

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Keeping in mind $x_i = x \bmod m_i$

Consider the product state:

$$|x_{\text{RNS}}\rangle = \bigotimes_i |x_i\rangle$$

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Apply the following QFT-ish unitary to each:

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Takeaway 1: We just did the QFT modulo L!

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Takeaway 2: Peter Shor is a really smart guy

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Thank you!

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