



Workshop: quantum algorithms for asymmetric cryptography

Greg Meyer ¹

¹MIT

July 9, 2026

Most important open problem: why we are all here

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Let me know if you have ideas; we can vote on Friday.

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Let me know if you have ideas; we can vote on Friday.

We're on a timeline to figure out a name because it will be dead soon...

Why we are actually here: quantum attacks are coming

Google:

Quantum frontiers may be closer than they appear

Mar 25, 2026
2 min read

We're setting a timeline for post-quantum cryptography migration to 2029.

Cloudflare:

Cloudflare targets 2029 for full post-quantum security

2026-04-07

Why we are actually here: quantum attacks are coming

US government:



45 47

PRESIDENTIAL ACTIONS

SECURING THE NATION AGAINST ADVANCED CRYPTOGRAPHIC ATTACKS

Executive Orders | June 22, 2026

- (ii) transition all HVAs and high impact systems to use PQC for key establishment by December 31, 2030;
- (iii) transition all HVAs and high impact systems to use PQC for digital signatures by December 31, 2031; and

Why we are actually here: quantum attacks are coming

As experts in those quantum attacks, what do we do about it?

Goals of the workshop

Roadmap

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Landscape of vulnerable cryptography

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Some history: how did we get here?

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Workshop goals

Discuss the *operational* status of quantum cryptanalysis



The screenshot shows the Simons Institute website. At the top left is the Simons Institute logo, which consists of a stylized grid of blue squares next to the text "SIMONS INSTITUTE for the Theory of Computing". To the right of the logo are two links: "Programs & Events" and "Participate". Below the navigation bar, the main heading "Quantum Circuits and Algorithms for Cryptography" is displayed in a large, bold, black font. Underneath the heading, there are three lines of text: "Program" followed by a link to "Summer Cluster on Quantum Computing 2026", "Location" followed by "Calvin Lab auditorium", and "Date" followed by "Thursday, July 9 – Friday, July 10, 2026". At the bottom of the section, there are two buttons: a dark grey button with a calendar icon and the text "Add to Calendar" with a dropdown arrow, and a white button with the text "Back to calendar".

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- What *ideas/tricks* are important for the currently best circuits?

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Don't be afraid to interrupt with questions, ideas, etc.!

Workshop schedule

	Thursday	Friday
9:30-10:30	Speaker: Greg Meyer	Speaker: Craig Gidney
11:00-12:00	Speaker: Martin Ekerå	Speaker: André Schrottenloher
2:00-3:00	Speaker: Seyoon Ragavan	Panel 2: <i>Responsible disclosure</i>
3:30-5:00	Panel 1: <i>Open directions</i>	Lightning talks/discussion

Panel 1: Open directions

- Martin Ekerå (Swedish NCSA and KTH)
- Craig Gidney (Google)
- André Schrottenloher (INRIA)

Panel 2: Responsible disclosure

- Ryan Babbush (Google)
- Rolfe Schmidt (Signal)
- Martin Ekerå (Swedish NCSA and KTH)
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- A 5-10 minute talk (whiteboard or otherwise)

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- Impromptu presentations also OK

Roadmap

Goals of the workshop

Landscape of vulnerable cryptography

Some history: how did we get here?

Abstract computational problems

Discrete logarithm

Consider a multiplicative group $\mathbb{G}$
with generator g .

Given a group element $x \in \mathbb{G}$,
find d such that $g^d = x$.

Integer factorization

Consider an integer N .

Find its prime decomposition.

Vulnerable concrete problems used in cryptography

Elliptic curve discrete log (ECDLP)

For an elliptic curve group $\mathbb{G}$,
consider two curve points
 g and $x = [d]g$.

Find d .

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Finite field discrete log

(FFDLP)

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(typically a prime).
Given positive integers
 g and $x \equiv g^d \pmod{p}$:

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RSA integer factorization

(IFP)

Consider an integer $N = pq$,
where p, q are primes of
equal bit length.

Find p and q .

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Note: cryptosystems rely on slightly stronger hardness assumptions

Comparing the vulnerable problems

Note: this entire slide is my qualitative assessment, in July 2026

IFP = integer factorization; ECDLP = elliptic curve discrete log; FFDLP = finite field discrete log

Attention given

How widely used?

Quantum cost?

Comparing the vulnerable problems

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1 = most expensive

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1. IFP	1. ECDLP	1. IFP
2. ECDLP	2. IFP	2. ECDLP
3. FFDLP	3. FFDLP	3. FFDLP

Takeaway: factoring gets by far the *most attention*, but not seemingly for any *good reason*.

Quantum cost: rough overview

Elliptic curve discrete log (ECDLP)

Given $x = [d]g$, find d .

$$n = \lceil \log_2 |\mathbb{G}| \rceil$$

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For $n = 256$:

- Qubits: ~ 1200
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Qubits: $m + o(m) + O(\log n)$

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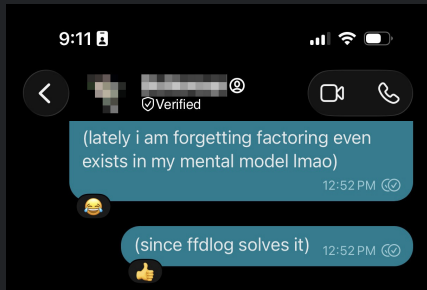
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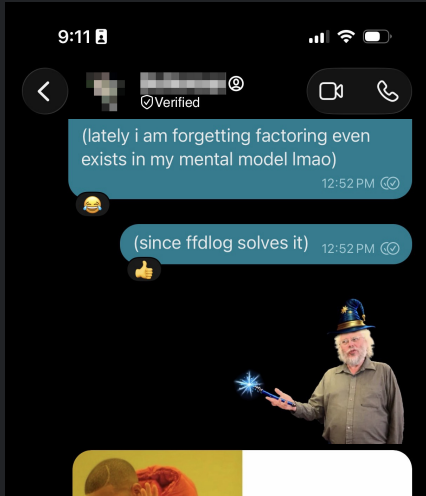
A window into my scientific process



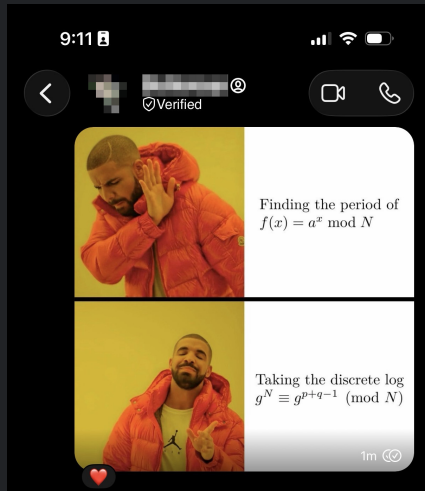
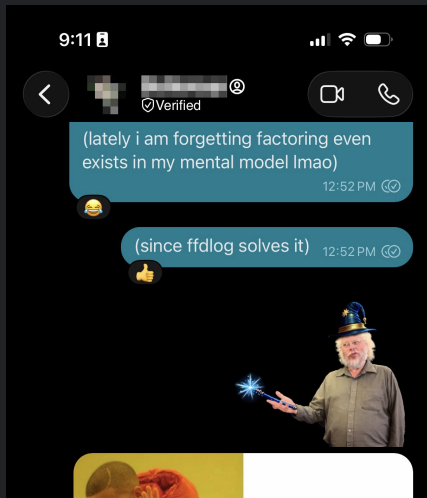
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Why discrete log is the thing to focus on

Factoring RSA numbers reduces to discrete log;

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furthermore, doing it this way is actually **more efficient**.

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Factoring RSA numbers reduces to discrete log;
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→ You will hear about this reduction (and more) from Martin
during his **11 AM-12 PM talk today!**

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RSA integer factorization (IFP)

Given $N = pq$, find p and q .

$$n = \lceil \log_2 N \rceil$$

$\Leftarrow$ FFDLP with params.

$n = \lceil \log_2 N \rceil, m = n/2$

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Given $N = pq$, find p and q .

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$\Leftarrow$ FFDLP with params.

$n = \lceil \log_2 N \rceil$, $m = n/2$

For $n = 2048$:

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Quantum cost: rough overview

Elliptic curve discrete log (ECDLP)

Given $x = [d]g$, find d .

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Qubits: $\sim 4.4n + O(\sqrt{n})$

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For $n = 256$:

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Finite field discrete log (FFDLP)

Given $x \equiv g^d \pmod{p}$ find d .

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Qubits: $m + o(m) + O(\log n)$

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For $n = 2048$, $m = 225$:

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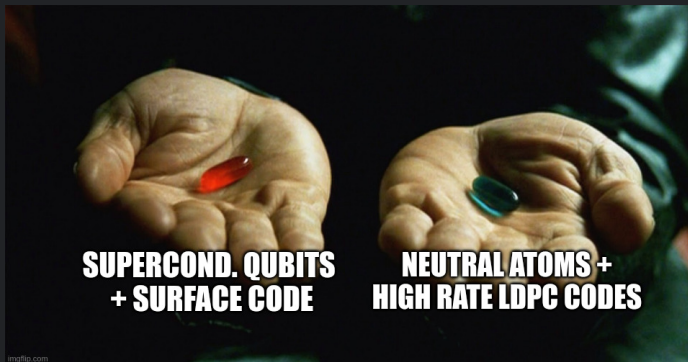
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Take a second look at old constructions under new heuristics?

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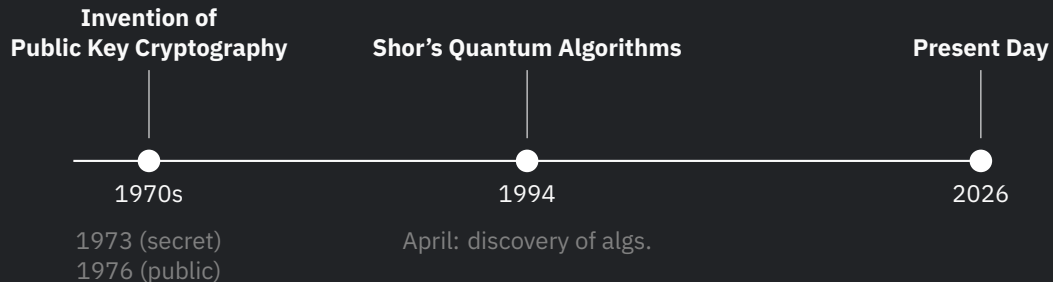
Roadmap

Goals of the workshop

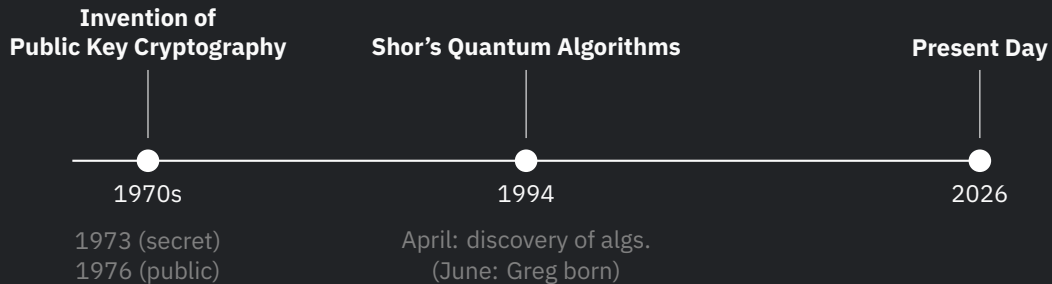
Landscape of vulnerable cryptography

Some history: how did we get here?

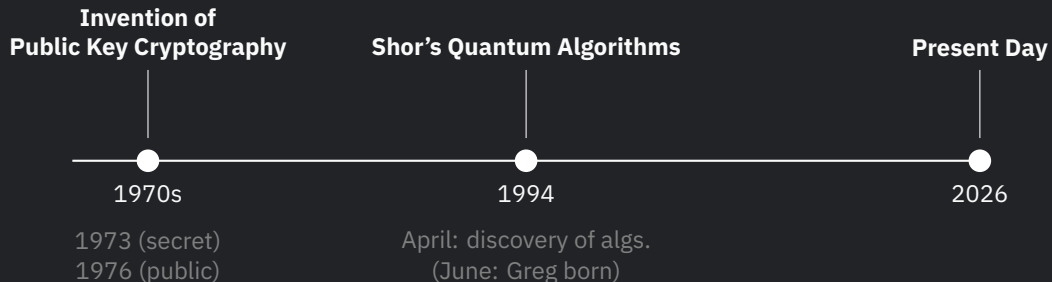
Timeline of public-key cryptography



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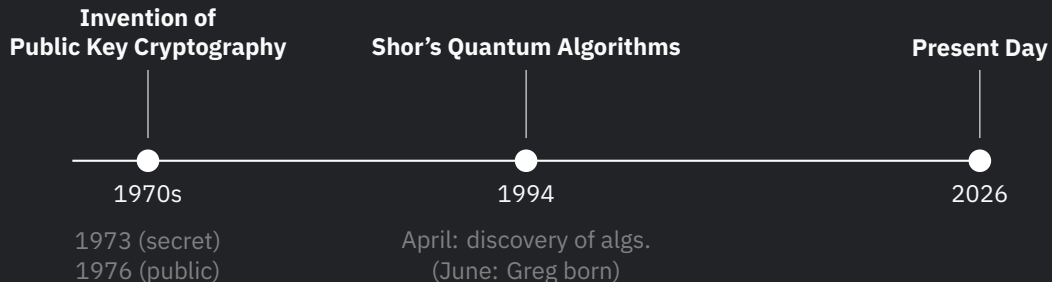


Timeline of public-key cryptography



Happy 32nd birthday to Shor's algorithms! 🎉

Timeline of public-key cryptography



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Shor's discovery was closer to the invention of public-key cryptography than it is to the present day

Timeline of quantum circuits for discrete log + factoring

Shor (1994)

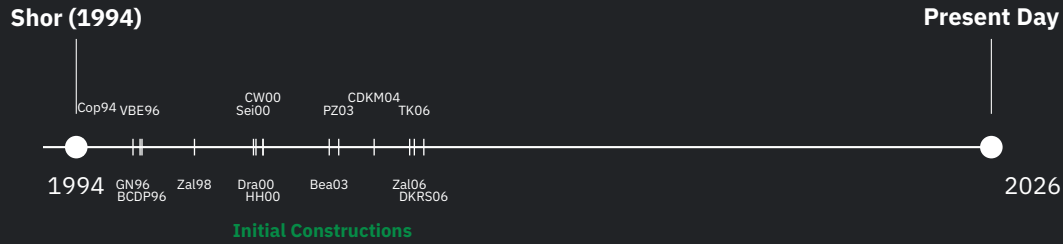
Present Day



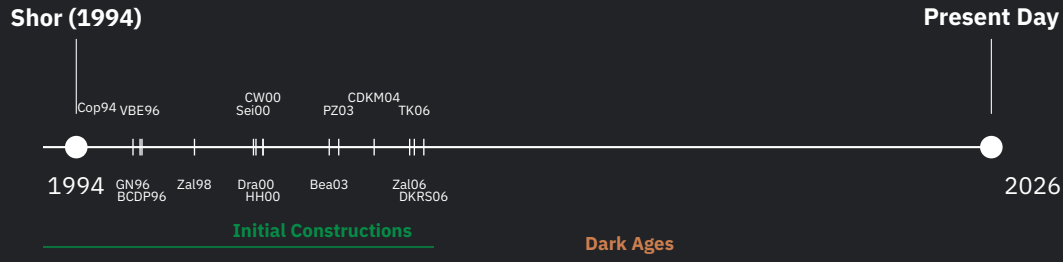
1994

2026

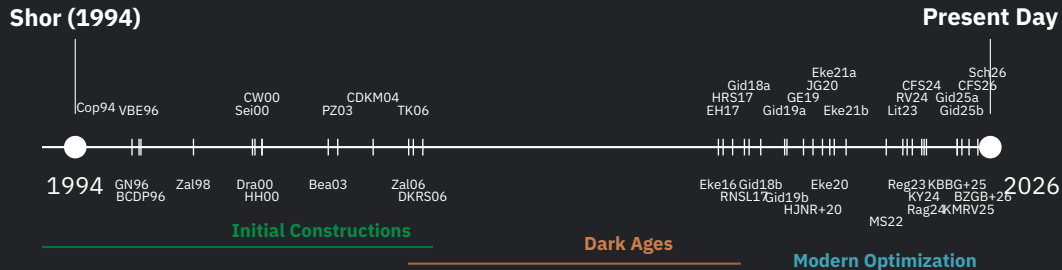
Timeline of quantum circuits for discrete log + factoring



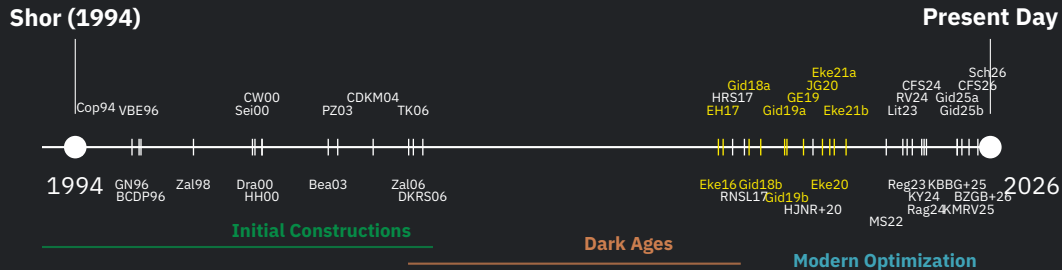
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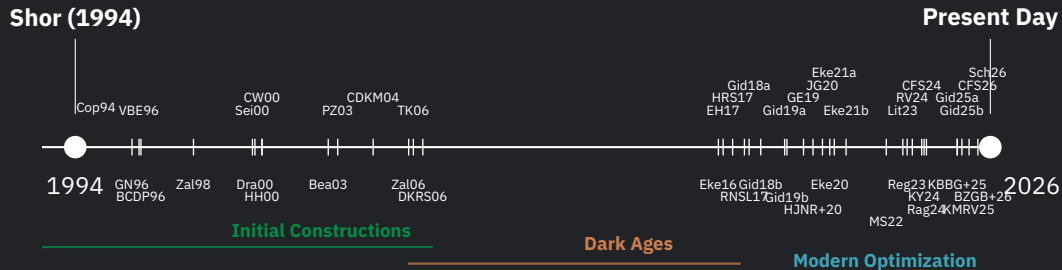
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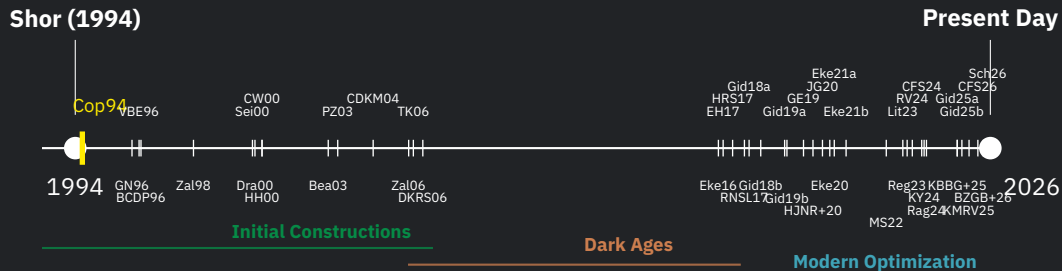
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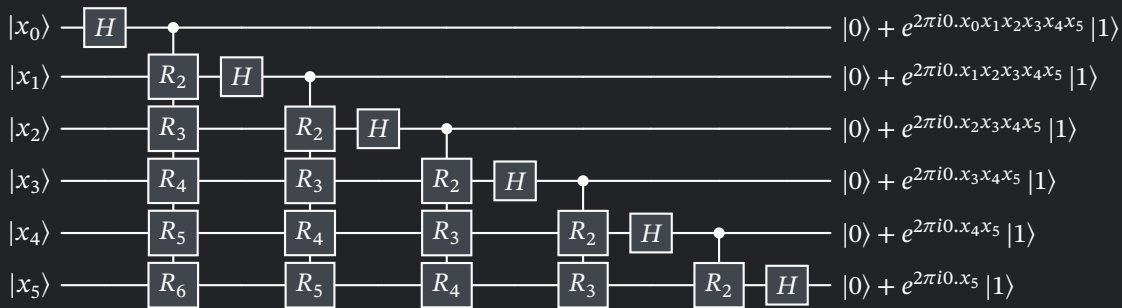


Timeline of quantum circuits for discrete log + factoring



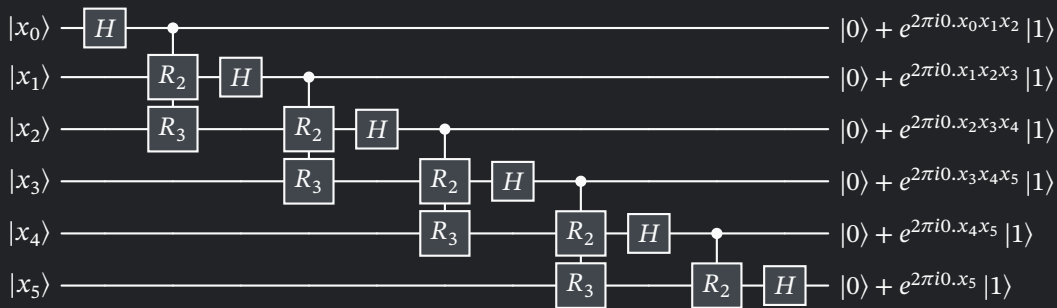
Coppersmith '94: approximate quantum Fourier transform modulo 2^n

This is the “standard” QFT construction



Coppersmith '94: approximate QFT_{2^n}

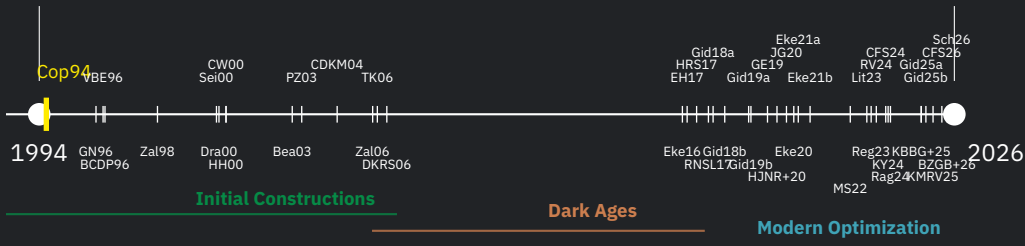
Approximate QFT: drop small rotations!



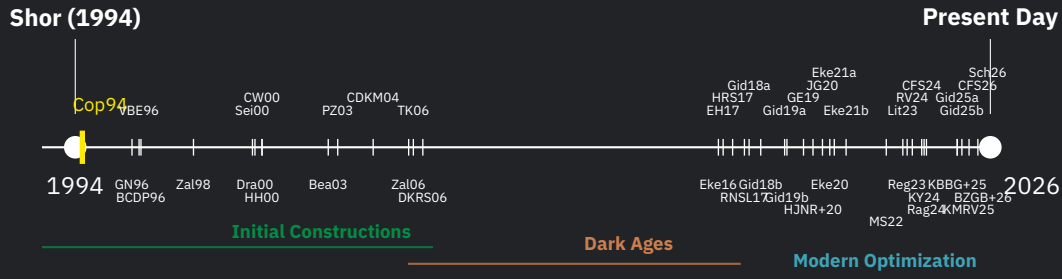
Timeline of quantum circuits for discrete log + factoring

Shor (1994)

Present Day



Timeline of quantum circuits for discrete log + factoring



Peter Shor didn't have the now-standard QFT_{2^n} construction for his original paper!

Peter Shor's original quantum Fourier transform (QFT)

Crucial question: can we efficiently perform the Fourier transform on the amplitudes of an n -qubit quantum state?

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 - This is possible via a **Residue Number System (RNS)**.

Aside: residue number systems (RNS)

A funky and neat alternative to binary representation!

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A funky and neat alternative to binary representation!

Note: You will hear a lot more about RNS from André tomorrow at 11 AM!

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1. Pick a bunch of co-prime integers $\{m_i\}$, each of length $O(\log n)$

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Fact: (*Chinese Remainder Theorem*)

If $x < L$:

$$\sum_i x_i F_i \bmod L = x$$

(where F_i are some pre-computable integers that don't depend on x).

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Takeaway 1: We just did the QFT modulo L !

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Takeaway 2: Peter Shor is a really smart guy



INTELLIGENCE COMMUNITY
POSTDOCTORAL RESEARCH
FELLOWSHIP PROGRAM



SIMONS
INSTITUTE
for the Theory of Computing

Thank you!

GDKM was supported by an appointment to the Intelligence Community Postdoctoral Research Fellowship Program at MIT administered by Oak Ridge Institute for Science and Education (ORISE) through an interagency agreement between the U.S. Department of Energy and the Office of the Director of National Intelligence (ODNI).